A Dynamic Interaction Index Based on Set Point Transmittance

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Most chemical processes are multivariable in nature, that is, they are characterized by multiple inputs (manipulated variables) and multiple outputs (controlled variables). A number of investigators have developed measures of interaction that allow a control system designer to determine *a priori* the proper input-output pairing for a set of single-input/single-output (SISO) controllers. The purpose of this paper is to extend an approach developed by Tung and Edgar (1981) to examine the dynamic and static interactions among process inputs and outputs for the purpose of selecting the proper pairing for control system design. We also place this extension in context with recent papers by Jensen et al. (1986) and Friedly (1984), and present a more general framework for low-frequency interaction analysis of multivariable systems. This work is based on the premise that all multivariable feedback control systems with integral action, operating on a square nonsingular process system, will have the same steady state manipulated variable change for a given set point change.

Interaction Analysis

Many techniques have been developed to aid the control system designer in the pairing of manipulated and controlled variables. Bristol (1966) developed the relative gain array (RGA) to compare open- and closed-loop steady state sensitivities between input and output variables. The RGA is a very powerful tool and is the most often cited method for control system synthesis. Grosdidier et al. (1985) have provided a rigorous justification for its popularity. There are many cases, however, where steady state information is not enough to provide an acceptable control system design and a method based on system dynamics is needed.

Witcher and McAvoy (1977) extended the RGA to the frequency domain to measure dynamic interaction. Tung and Edgar (1981), Gagnepain and Seborg (1982), Lau et al. (1985), Jensen et al. (1986), Economou and Morari (1986), Grosdidier and Morari (1986), and Mijares et al. (1986) have all developed dynamic measures of interaction. Friedly (1984) has discussed conditions in which the RGA based on steady state analysis fails.

Tung and Edgar (1981) have developed a technique for examining dynamic interactions for linear systems which includes the steady state RGA as a special case. This interaction measure accounts for the direct and parallel transmittances of a set point change in y_i to output y_i, and can be extended to account for the transmittance of a set point change in y_j to output y_i.

Let a linear dynamic system be described by

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

Consider a set point change from 0 to y^o and let the required control change be $u = u^o$. Then

$$u^o = [C(-A)^{-1}B]^{-1}y^o \tag{3}$$

With zero initial condition, the output response is

$$\begin{aligned} y(s) &= Cx(s) \\ &= C(sI - A)^{-1}B[C(-A)^{-1}B]^{-1}y^o/s \\ y(s) &= \Phi(s)\Phi(0)^{-1}y^o/s \end{aligned} \tag{4}$$

where $\Phi(s)$ is the transfer function matrix, that is,

$$y_i(s) = \sum_{j=1}^{m}\left[\sum_{k=1}^{m}\Phi_{i,k}(s)\Gamma_{k,j}\right]y_j^o/s \tag{5}$$

where $\Phi_{i,k}(s)$ and $\Gamma_{k,j}$ are elements of the $C(sI - A)^{-1}B$ matrix and $[C(-A)^{-1}B]^{-1}$ matrix $[\Phi^{-1}(0)]$, respectively.

Tung and Edgar considered a step change in y_i^o only, yielding the following response:

$$y_i(s) = \sum_{k=1}^{m}\Phi_{i,k}(s)\Gamma_{k,i}y_i^o/s \tag{6}$$

They then defined the interaction index as

$$\alpha_{i,k}(s) = \Phi_{i,k}(s)\Gamma_{k,i}/s \quad (7)$$

However, it should be defined as

$$\alpha_{i,k}(s) = \Phi_{i,k}(s)\Gamma_{k,i} \quad (8)$$

in order to be viewed properly in the frequency domain. $\alpha_{i,k}$ is the transmittance from set point i to output i through the kth controller. At steady state the Tung and Edgar interaction index reduces to the relative gain $\alpha_{i,k}(0) = \lambda_{ik}(0)$.

At this point it is important to clarify some misunderstandings in the literature concerning the methodology developed by Tung and Edgar (1981). Jensen et al. (1986) comment that:

1. A state space model is required, and
2. A statically decoupled system is assumed before the dynamic interaction analysis is performed

Both of these statements are false. The interaction index can be viewed in either the frequency or the time domain, although most engineers have a clearer understanding of the time-domain response curves. No assumption is made regarding a statically decoupled system. Rather, a term representing the inverse of the steady state model appears, due to the fact that the control action at steady state requires the model inverse.

In the next section we extend the method to consider the effect of a set point change in one loop on the other loops. This extension, the set point transmittance index, is important because it can identify potential dynamic interaction problems in a closed-loop system that are not predictable by other RGA-based measures.

Set Point Transmittance Index

In Eq. 5 Tung and Edgar assumed a set point change in y_i^o only. This assumption only treats the direct and parallel transmittances in a process system, Figure 2. A more general development involves a set point change in y_j^o. Then $\beta_{i,k,j}$ can be defined as the set point transmittance index

$$\beta_{i,k,j}(s) = \Phi_{i,k}(s)\Gamma_{k,j} \quad (9)$$

representing the transmittance from set point j to output i through controller k. For 2 × 2 systems, the following relationships hold for output 1:

$$\begin{aligned}
\alpha_{1,1}(s) &= \Phi_{1,1}(s)\Phi_{2,2}(0)/\det\Phi(0) \\
\alpha_{1,2}(s) &= -\Phi_{1,2}(s)\Phi_{2,1}(0)/\det\Phi(0) \\
\beta_{1,1,2}(s) &= -\Phi_{1,1}(s)\Phi_{1,2}(0)/\det\Phi(0) \\
\beta_{1,2,2}(s) &= \Phi_{1,2}(s)\Phi_{1,1}(0)/\det\Phi(0) \quad (10)
\end{aligned}$$

and for output 2:

$$\begin{aligned}
\alpha_{2,1}(s) &= -\Phi_{2,1}(s)\Phi_{1,2}(0)/\det\Phi(0) \\
\alpha_{2,2}(s) &= \Phi_{2,2}(s)\Phi_{1,1}(0)/\det\Phi(0) \\
\beta_{2,1,1}(s) &= \Phi_{2,1}(s)\Phi_{2,2}(0)/\det\Phi(0) \\
\beta_{2,2,1}(s) &= -\Phi_{2,2}(s)\Phi_{2,1}(0)/\det\Phi(0) \quad (11)
\end{aligned}$$

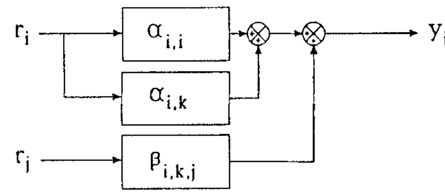

Figure 1. Representation of dynamic interaction index and set point transmittance index.

These terms may also be viewed in the time domain

$$\Delta_{i,k} = L^{-1}\{\alpha_{i,k}(s)/s\} \quad (12)$$

$$\gamma_{i,k,j}(t) = L^{-1}\{\beta_{i,k,j}(s)/s\} \quad (13)$$

where a step change in set point has been assumed.

Properties of β include:

$$\sum_{k=1}^{m} \beta_{i,k,j}(0) = 0.0 \quad \text{for } i \neq j$$

$$\sum_{k=1}^{m} \beta_{i,k,j}(0) = \sum_{k=1}^{m} \alpha_{i,k}(0) = 1.0 \quad \text{for } i = j$$

Unlike the relative gain array or the Tung and Edgar (T-E) dynamic interaction index, the set point transmittance index is not independent of scaling. The transmittances represented by the T-E dynamic interaction index, Eq. 8, and the set point transmittance index, Eq. 9, are shown in Figure 1. Note the similarity to the classification of transmittances according to Jensen et al. (1986), shown here in Figure 2.

The similarity of the β set point interaction terms with Friedly's (1984) gain ratio (r_{ij}) should be noted. For example:

$$r_{12} = \Phi_{1,2}(0)/\Phi_{2,2}(0) \quad (14)$$

$$\beta_{1,2,2}(s) = [\Phi_{1,2}(s)/\Phi_{2,2}(0)]\lambda_{22} \quad (15)$$

Friedly based his development on closed-loop proportional feedback control in the steady state, and showed that even if the steady state RGA indicated little interaction, it was possible to have the second loop interact with the first if the ratio r_{12} was sufficiently large. In this case $\beta_{1,2,2}(0) \approx r_{12}$ since $\lambda_{22} \approx 1.0$.

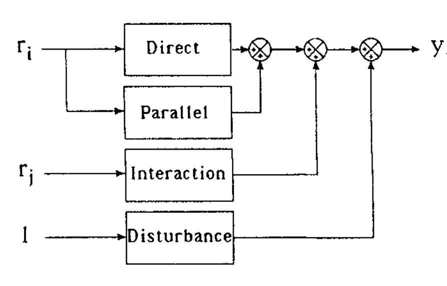

Figure 2. Transmittances, as classified by Jensen et al. (1986).

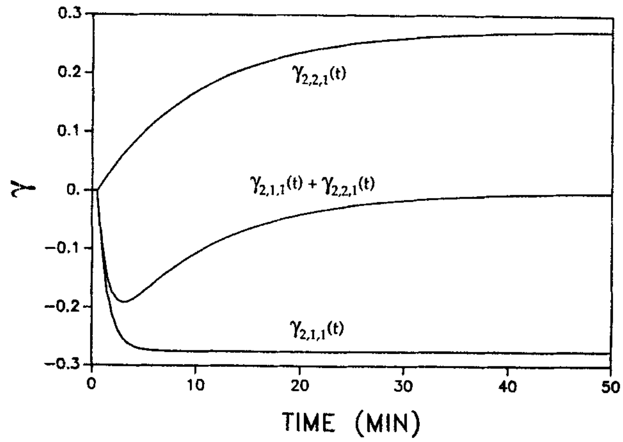

Figure 3. Application example: analysis by Eq. 13.

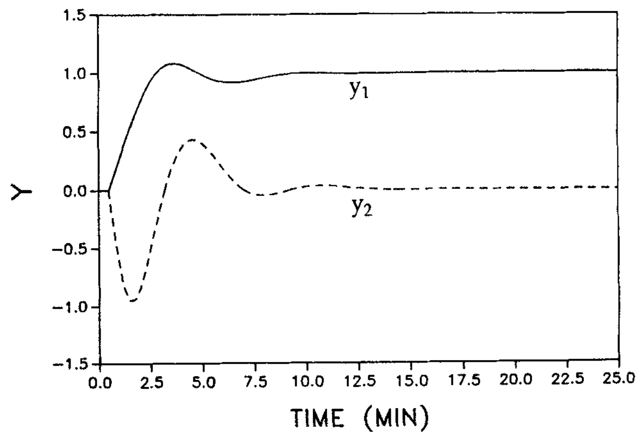

Figure 5. Application example: set point change in y_1.

Application Example

Consider the following transfer function model

$$G(s) = \begin{bmatrix} \frac{1.0\,e^{-0.5s}}{10s+1} & \frac{0.3\,e^{-0.5s}}{10s+1} \\ \frac{-0.3\,e^{-0.5s}}{s+1} & \frac{1.0\,e^{-0.5s}}{10s+1} \end{bmatrix}$$

with a relative gain array

$$\Lambda(0) = \begin{bmatrix} 0.92 & 0.08 \\ 0.08 & 0.92 \end{bmatrix}$$

The time domain set point transmittance indices, Eq. 13, are shown in Figures 3 and 4. Figure 3 predicts that a set point change in y_1 will disturb y_2 because of the quick dynamics that propagate through u_1. Figure 4 shows that a set point change in y_2 will not disturb y_1 because of the equivalent dynamics that propagate through u_1 and u_2. Notice that since the transmittance from set point y_1 to output y_2 is dynamic in nature, it cannot be predicted by Friedly's gain ratio method.

The closed-loop characteristics predicted by the set point transmittance indices are shown clearly in Figures 5 and 6. The improved IMC-PI tuning rules developed by Rivera et al. (1986) were used to determine the controller tuning parameters, $k_c = 5.125$ and $\tau_I = 10.25$, for each control loop.

Conclusions

The general framework for interaction analysis that we have presented is a good predictor of interaction at low frequencies for the following reason. Knowledge of the required control action u^o necessary to correct a set point change y^o is implicit in the derivation of both the Tung and Edgar interaction index and the set point transmittance index. Since any stable control strategy with integral action must employ this u^o, we have a reliable indicator of low-frequency interaction. From a practical point of view, the steady state case is more important than the high-frequency case, since controllers are not tuned for high-frequency control action.

In this work Tung and Edgar's interaction index was incorporated into a more general framework for interaction analysis with the development of the set point transmittance index. The set point transmittance predicts control loop interactions that cannot be predicted by the RGA methods. This type of interaction is also predicted by the IMC interaction measures (Economou and Morari, 1986).

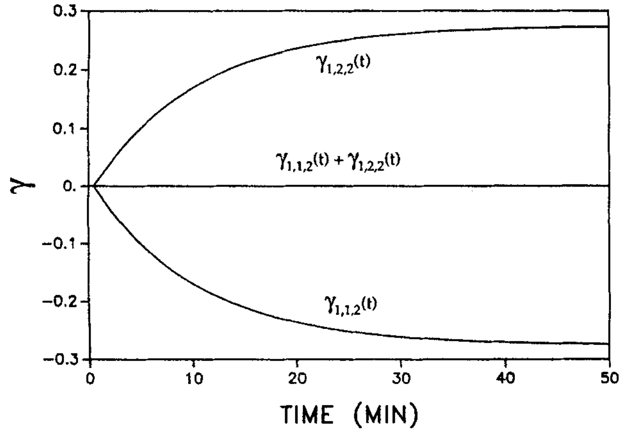

Figure 4. Application example: analysis by Eq. 13.

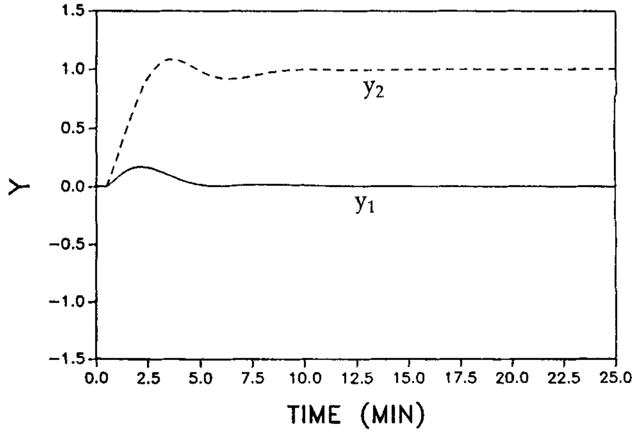

Figure 6. Application example: set point change in y_2.

Notation

A – state matrix
B – control matrix
C – measurement matrix
det – determinant
G – gain matrix
I – indentity matrix
l – load input
L^{-1} – inverse Laplace transform
m – dimension of y and u
n – dimension of x
RGA – relative gain array
s – Laplace transform variable
t – time
u – manipulated variable vector
x – state variable vector
y – output variable vector

Greek letters

$\alpha_{i,j}$ – T-E interaction index
$\beta_{i,k,j}$ – set point transmittance index
$\Delta_{i,k}$ – time-domain T-E interaction index
$\gamma_{i,k,j}$ – time-domain set point transmittance index
$\Gamma_{i,j}$ – element of inverse of steady state gain matrix
$\lambda_{i,j}$ – element of RGA
$\phi_{i,j}$ – element of transfer function matrix

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Manuscript received Mar. 9, 1987, and revision received Jan. 5, 1988.